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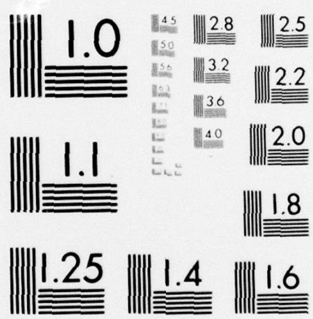

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This paper presents a method for the realization of non-linear estimators based on spline interpolation. The difference of a monomial and its interpolating spline forms a monospline and then a quadrature formula is induced. When the knots of the monospline at which the conditional density is discretized are allowed to vary, a class of optimal quadrature formulas is obtained. To find the monospline with optimal knots a set of non-linear algebraic equations must be solved. If the symmetry property of the monospline is applied, the order of the non-linear equations can be reduced by about one-half. An iteration (continued)

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IMPLEMENTATION OF NON-LINEAR ESTIMATORS USING MONOSPLINE

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Abstract

This paper presents a method for the realization of non-linear estimators based on spline interpolation. The difference of a monomial and its interpolating spline forms a monospline and then a quadrature formula is induced. When the knots of the monospline at which the conditional density is discretized are allowed to vary, a class of optimal quadrature formulas is obtained. To find the monospline with optimal knots a set of non-linear algebraic equations must be solved. If the symmetry property of the monospline is applied, the order of the non-linear equations can be reduced by about one-half. An iteration scheme of Newton type is introduced to solve the monospline. The quadrature formula associated with this monospline has the so-called positivity property which is essential in the practical implementation of non-linear recursive estimators.

1. Introduction

The implementation problem for non-linear estimators of the state in a discrete time dynamical system employs a recursive algorithm arising from Bayes Law. Many numerical approaches for this implementation problem have been developed during the past few years. These include curve fitting the conditional density [1][2] and the quadrature approximation to the necessary integrations [3][4][5]. The essential step in obtaining the conditional density of the state, given the observations, is integration.

In this paper, a method using quadrature formulas related to monosplines with optimal knots is discussed.

Consider a discrete time dynamical system given by

$$\underline{x}_k+1 = \underline{f}(\underline{x}_k) + \underline{w}_k \qquad \underline{x}_k, \underline{w}_k \in R^n \tag{1}$$

with measurement

$$\underline{y}_k = \underline{h}(\underline{x}_k) + \underline{v}_k \qquad \underline{y}_k, \underline{v} \in R^m \tag{2}$$

The noise sequences are assumed to be independent Guassian random vectors with zero mean. If the p.d.f. of the initial state is assumed to be known, the conditional density function of the state, given the observations, can be computed by the following recursive equations [6]

$$\phi(\underline{x}_k|Y_k) = \frac{u(\underline{y}_k|\underline{x}_k)p(\underline{x}_k|Y_{k-1})}{\int u(\underline{y}_k|\underline{x}_k)p(\underline{x}_k|Y_{k-1})d\underline{x}_k} \tag{3}$$

and

$$p(\underline{x}_k+1|Y_k) = \int \tau(\underline{x}_{k+1}|\underline{x}_k)(\underline{x}_k|Y_k)d\underline{x}_k \tag{4}$$

where ϕ is filtering density, u is measurement density, τ is transition density and p is prediction density. This paper attempts to approximate the necessary integrations in (3) and (4) by using the quadrature formula derived from monospline.

2. Spline Interpolation, Monosplines and Corresponding Quadrature Formulas

Let s(x) be a spline function of degree 2m-1 with distinct knots $x_1, x_2, \ldots, x_n$ in [-1,1] and k-fold knots at -1 and 1, where $0<k<m\le n+2k$. Then if a monomial of degree 2m, $x^{2m}/(2m)!$, is interpolated by s(x) in [-1,1], the difference between these two functions becomes a monospline

$$H(x) = \frac{x^{2m}}{(2m)!} - s(x)$$

$$= \frac{x^{2m}}{(2m)!} - \sum_{i=1}^{2m} \alpha_i \frac{x^{i-1}}{(i-1)!} - \sum_{j=1}^{n} \beta_j \frac{(x-x_j)_+^{2m-1}}{(2m-1)!} \tag{5}$$

The spline interpolation asserts that

The results presented in this paper were obtained with the partial support of the Air Force Office of Scientific Research under Grant AFOSR75-2828.

$$H(x_j) = 0 \qquad j=1, 2, \ldots, n$$

$$H^{(i)}(\pm 1) = 0 \qquad i=1, 1, \ldots, k-1 \qquad (6)$$

$$H^{(m+p)}(\pm 1) = 0 \qquad p=0, 1, \ldots, m-k-1$$

Let $K(x) = H^{(m)}(x) = \frac{x^m}{m!} - s^{(m)}(x)$ (7)

Then the orthogonality property of spline interpolation implies that [7]

$$\int_{-1}^{1} [K(x)]^2 = \text{minimum} \qquad (8)$$

Since $K(x) \varepsilon C^{m-2}$, for any function $f(x) \varepsilon C^m[-1,1]$, we can employ integration by parts to get

$$\int_{-1}^{1} f(x)dx = \sum_{j=1}^{n} A_j f(x_j)$$

$$+ \sum_{i=0}^{k-1} [B_i f^{(i)}(-1) + D_i f^{(i)}(+1)]$$

$$+ (-1)^m \int_{-1}^{1} K(x) f^{(m)}(x)dx \qquad (9)$$

where $\{A_j\}$, $\{B_i\}$ and $\{D_i\}$ are the coefficients of the derived quadrature formula which relate to $K(x)$ by

$$A_j = K^{(m-1)}(x_{j-}) - K^{(m-1)}(x_{j+}) = \beta_j$$

$$j=1, 2, \ldots, n \qquad (10)$$

$$B_i = (-1)^{i+1} K^{(m-i-1)}(-1) \quad i=0, 1, \ldots, k-1 \quad (11)$$

$$D_i = (-1)^i K^{(m-i-1)}(+1) \quad i=0, 1, \ldots, k-1 \quad (12)$$

$K(x)$ is then a kernel of the derived quadrature formula which has remainder

$$Rf = (-1)^m \int_{-1}^{1} K(x) f^{(m)}(x)dx \qquad (13)$$

For the case $2 \le m$, $0 \le k \le m \le 2n+2k$ and $\{x_j\}$ are double knots, a set of additional condition is induced,

$$H'(x_j) = 0 \qquad j=1, 2, \ldots, n \qquad (14)$$

and (9) has additional term $\sum_{j=1}^{n} \xi_j f'(x_j)$ added to its right hand side. ξ_j is given by

$$\xi_j = K^{(m-2)}(x_{j+}) - K^{(m-2)}(x_{j-}) \qquad (15)$$

$$j=1, 2, \ldots, n$$

If the knots are allowed to vary, a set of optimal knots can be obtained which cause the ξ_j 's to vanish. This induces so-called optimal property to the Quadrature Formulas (9), [8].

3. Numerical Solution for the Monospline with Optimal Knots

Substituting the conditions (6) and (14) into (5), 2m+2n non-linear algebraic equations are obtained in the 2m+2n unknowns, $\{\alpha_i\}$, $\{\beta_j\}$ and $\{x_j\}$. It can be shown [8] that a symmetry property of the monospline can be employed to reduce the number of equations and unknowns to m+n when n is even or m+n+1 when n is odd. This eliminates even terms of $\{\alpha_i\}$.

Let $\underline{x} = [x_1 \; x_2 \; \ldots \; x_q]^T$ (16)

and $\underline{y} = [\alpha_1 \; \alpha_3 \; \ldots \; \alpha_{2m-1}, \beta_1, \beta_2, \ldots, \beta_q]^T$ (17)

where q=n/2 when n is even or q=n+1/2 when n is odd. Then the conditions (6) and (14) induce the following equations

$$A(\underline{x})\underline{y} = \underline{z}(\underline{x}) \qquad A: (m+q)x(m+q) \qquad (18)$$

and $B(\underline{x})\underline{y} = \underline{w}(\underline{x}) \qquad B: qx(m+q)$ (19)

Let $\underline{\eta}$ represent all the unknowns

$$\underline{\eta} = \begin{bmatrix} \underline{y} \\ \underline{x} \end{bmatrix} \qquad (20)$$

and combine (18) and (19). We get

$$\underline{f}(\underline{\eta}) = \begin{bmatrix} A(\underline{x})\underline{y} - \underline{z}(\underline{x}) \\ B(\underline{x})\underline{y} - \underline{w}(\underline{x}) \end{bmatrix} = 0 \qquad (21)$$

The Jacobian of (21) is computed by taking partial derivatives of $\underline{f}(\underline{\eta})$ with respect to $\underline{\eta}$. Since (21) has a unique solution, the inverse of this Jacobian exists. Partitioning the inverse Jacobian, we get

$$J^{-1} = \begin{bmatrix} A^I & E^I \\ B^I & F^I \end{bmatrix} \qquad (22)$$

where A^I: (m+q)x(m+q), B^I: qx(m+q), E^I: (m+q)xq and F^I: qxq.

Since (18) is linear in $\underline{y}$ if $\underline{x}$ is given, (21) can be simplified by computing $\underline{y}$ from $\underline{x}$. Then we get

$$\underline{f}(\underline{\eta}) = \begin{bmatrix} 0 \\ \underline{g}(\underline{x}) \end{bmatrix} \qquad (23)$$

where

$$\underline{g}(\underline{x}) = B(\underline{x})A^{-1}(\underline{x})\underline{z}(\underline{x}) - \underline{w}(\underline{x}) \qquad (24)$$

Then a modified Newton's iteration equation (9) can be used to obtain $\underline{x}$,

$$\underline{x}_{k+1} = \underline{x}_k - t_k F_k^I \underline{g}(\underline{x}_k) \tag{25}$$

where t_k is a scalar factor chosen to insure the convergence of the iteration process, e.g.

$$||\underline{g}(\underline{x}_{k+1})|| < ||\underline{g}(\underline{x}_k)|| \tag{26}$$

where k is the iteration parameter.

4. Optimal Quadrature Formulas for Updating Procedure

In order to maintain the positivity of the discretized density function in the recursive equations only those quadrature formulas with k=0 or k=1 and all positive weights are useful for the evolution of conditional densities. The optimal quadrature formulas derived from the monospline numerically computed in the previous section have all positive weights and require no derivatives of the integrand. Hence, O.Q.F.'s are practically useful in non-linear estimation problems.

Numerical experiments indicate that the accuracy of the approximation by O.Q.F. can be improved by increasing m as predicted by theory. However, the O.Q.F. of high m is more sensitive to the truncation factor than those of low m. An O.Q.F. of low m may give better results if the evolving density is multimodal.

In the realization of the evolution of conditional densities, the concepts of floating grid and coordinate rotation are employed to efficiently represent the density functions [5]. The updating algorithm is designed so that the computational efficiencies of parallelism can be exploited for real time applications.

5. Conclusion

An approach to problems of non-linear estimation implementation using quadratures which are derived from monosplines with optimal knots is given. The quality of the approximation depends on the degree of monospline, the number of knots and the truncation factor. The approximation error is reduced by efficiently representing the density function in digital form.

The parameters in the optimal quadrature formulas are computed a priori, off line using simplifications of non-linear equations which specify them. Specifically, the separation of linear and non-linear parameters leads to a powerful and accurate iteration scheme to obtain the monospline with optimal knots.

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